Week 6 Activity: Exploring outcomes in the Lotka-Volterra competition model

In this activity, you will use an interactive app to explore how different aspects of the Lotka-Volterra competition model affect pairwise coexistence dynamics between species.

Section 1: Model review

Recall the model equations for the population dynamics of species 1 and 2:

$$\frac{dN_1}{dT} = r_1 N_1 (1 - \alpha_{11} N_1 - \alpha_{12} N_2)$$

$$\frac{dN_2}{dT} = r_2 N_2 (1 - \alpha_{21} N_1 - \alpha_{22} N_2)$$

1.1. What is the definition of each of the four α_{ij} parameters?

1.2. What is the carrying capacity (equilibrium population size) of Species 1 if it is growing alone in the system? Show your work to get to your answer.

> ***Note***: Recall that at equilibrium, $dN_1/dt = 0$, and that if species 1 is growing alone, $N_2 = 0$

Section 2: Model simulations

For the following questions, simulate the dynamics of this model with this interactive app: https://ecoevoapps.shinyapps.io/lotka_volterra_competition/.

Before you begin, click on the tab "Model terms in terms of absolute competition coefficients". Then, select the checkbox option to show "Zero net growth isoclines (ZNGIs)"

For this question, you will use the sliders to change values of α_{ij} in a way that satisfies a few different conditions. Then, for each parameter combination, explore what happens when you vary the values of N_1, N_2, r_1, and r_2.

2.1 Pick three sets of parameter combinations that satisfy the inequality:

$$\alpha_{11} > \alpha_{21} \text{ and } \alpha_{22} > \alpha_{12}$$

In at least one of the three sets, ensure that species 1 is on average more sensitive to competition than species 2: $\alpha_{11} + \alpha_{12} > \alpha_{21} + \alpha_{22}$

2.1.a. Parameter set 1:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

2.1.b. Parameter set 2:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

2.1.c. Parameter set 3:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

2.2 Pick three sets of parameter combinations that satisfy the inequality:

$$\alpha_{11} < \alpha_{21} \text{ and } \alpha_{22} < \alpha_{12}$$

In at least one of the three sets, ensure that species 1 is on average more sensitive to competition than species 2: $\alpha_{11} + \alpha_{12} > \alpha_{21} + \alpha_{22}$

2.2.a. Parameter set 1:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

2.2.b. Parameter set 2:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

2.2.c. Parameter set 3:

$\alpha_{11} =$ $\alpha_{21} =$

$\alpha_{12} =$ $\alpha_{22} =$

Explore different values of initial population size N_1 and N_2, and of intrinsic growth rate r_1 and r_2. What values of the parameters are you using, and what do you notice about the population dynamics?

3. From this exercise, what can you say about the conditions that allow coexistence in the Lotka-Volterra model?

4. The Lotka-Volterra competition model makes a lot of simplifying assumptions about nature. What are two assumptions implicit in this reductionist approach, and how might they differ from the complexity in nature? Do you think that including more complexity makes coexistence more or less likely in nature?